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(Residential Autonomous College under University of Calcutta)

B.A./B.SC. THIRD SEMESTER EXAMINATION, DECEMBER 2013

SECOND YEAR

Date : 23/12/2013 **MATH FOR ECONOMICS (General)**

Time : 11 am – 2 pm Paper : III Full Marks : 75

Group - A

Answer **any ten** questions of the following :

1. Show that the function $f(x,y) = |x| + |y|$ is continuous but not differentiable at (0,0). [5]

2. Find the directional derivative of $f(x,y) = x^2 + y + 1$ at (1,1) in the direction of the vector $\beta = (1,0)$. [5]

3. Consider the function $f(u,v) = \sqrt{|uv|}$, u(x) = v(x) = x. Can we apply chain rule to f to evaluate $\frac{df}{dx}$ at (0,0)? Give reason. [5]

4. Show that whatever the differentiable function ϕ is, the relationship $\phi(cx - az, cy - bz) = 0$ implies $a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c$. [5]

5. State and prove Euler's theorem on homogeneous function. [5]

6. Show that if $V = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then cos V is a homogeneous function and hence prove $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + \frac{1}{2}\cot v = 0$. [3+2]

7. Show that $y^3\cos x + y^2\sin^2 x = 7$ defines y as a function of x in some neighbourhood of $\left(\frac{\pi}{3}, 2\right)$. [5]

8. Let $u = \frac{x+y}{1-xy}$, $v = \tan^{-1}x + \tan^{-1}y$. Find $\frac{\partial(u,v)}{\partial(x,y)}$. Are they (i.e u and v) functionally related? If so find the relationship. [5]

9. Examine the existence of relative minima, relative maxima and saddle point for the function $f(x,y) = 2x^3 + 3xy^2 - 6x$. [5]

10. What do you mean by the convexity and concavity at a given point P on the plane curve y = f(x), with respect to a line not passing through P. [5]

11. Find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, where the parametres a, b are connected by $a^2 + b^2 = c^2$, where c is a constant. [5]

12. $f : [0,2] \to \mathbb{R}$ is defined by, $f(x) = \begin{cases} x + x^2, \text{when } x \text{ is rational} \\ x^2 + x^3, \text{when } x \text{ is irrational} \end{cases}$

examine whether f is Riemann integrable in [0,2]? [5]

13. Explain whether the relation $\int_a^b f'(x)dx = f(b) - f(a)$ is valid when $f(x) = \begin{cases} x^2\sin\frac{1}{x}, 0 < x \le 1 \\ 0 \quad , x = 0 \end{cases}$

Give reason in support of your answer. [5]

14. Examine the convergence of the improper integral $\int_{-1}^{0} \frac{e^{1/x}}{x^3} dx$ [5]

15. Compute the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. [5]

Group - B

Answer **any five** questions of the following :

16. Solve : $P^3 + 2P^2 - 2P = 0$, where $P \equiv \frac{dy}{dx}$. [5]

17. Solve : $y + Px = P^2x^4$, where $P \equiv \frac{dy}{dx}$. [5]

18. Solve : $(Px - y)(x - Py) = 2P$ [Hint : Substitute x^2 = u, y^2 = v and use clairaut's form] [5]

19. Solve : $\frac{d^2y}{dx^2} + 9y = 5x^2$. [5]

20. Solve using method of variation of parameters : $\frac{d^2y}{dx^2} + a^2y = \tan ax$, where a is a constant. [5]

21. Solve the following homogeneous differential equation : $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$. [5]

22. Solve by method of undetermined coefficients : $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = x^2e^x$. [5]

23. Solve : $x = y + P^2$, where $P \equiv \frac{dy}{dx}$. [5]